Reply to "Comment on 'Connection between the Burgers equation with an elastic forcing term and a stochastic process' "

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As pointed out by Garbaczewski in the Comment [Phys. Rev. E **74**, 028101 (2006)], the quantity $+\kappa^2 x$ should appear in Eq. (34) of the original paper [E. Moreau and O. Vallée, Phys. Rev. E **73**, 016112 (2006)], instead of $-\kappa^2 x$. Nevertheless, the main results are not affected by this fact, and, in particular, our methods of resolution and the conclusions exposed in our paper are unaffected by the error since the phrase "linear forcing term" is employed instead of "elastic forcing term."

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The transformation of the Comment under question [1], which makes it possible to transform Eq. (31) into Eq. (33) of Ref. [2], is valid either for term $-\kappa^2 x$ or for term $+\kappa^2 x$. The only mistake resides in the "−" sign appearing in Eq. (34) instead of the "+" sign. And our transformation [Eq. (35)] is independent of this mistake. Indeed, Eq. (35) is derived from Eqs. (31) and (33), which are both correct.

The error is that from the Ornstein-Uhlenbeck (OU) process [Eq. (27) of our paper]

$$\frac{dx}{dt} = -\kappa x + \sqrt{2\nu} b(t), \tag{1}$$

relations (29) to (33) of [2] lead in fact to the following Burgers equation:

$$\partial_t u + u\partial_x u = \nu \partial_{xx} u + \kappa^2 x, \tag{2}$$

instead of

$$\partial_t u + u\partial_x u = \nu \partial_{xx} u - \kappa^2 x; \tag{3}$$

as pointed out by Garbaczewski [1]. Consequently, we do not have an elastic term appearing in the Burgers equation, but only a (positive) linear term.

Our OU adapted Hopf-Cole transformation [Eq. (35)] is still valid and makes it possible to connect Eq. (2) to the probability density, the solution of the Fokker-Planck equation describing the Ornstein-Uhlenbeck process (1). Therefore, the stochastic study of this process is equivalent to solve a Burgers equation with a forcing term $\kappa^2 x$; the methods of resolution being described in the paper.

In conclusion, the Comment shows only that the erroneous sign in Eq. (34) implicates a bad term employed in the title with the following consequence: the relation [Eq. (35)] allows one in fact to connect an Ornstein-Uhlenbeck process to a Burgers equation with a linear (positive) forcing term.

[1] P. Garbaczewski, preceding Comment, Phys. Rev. E **74**, 028101 (2006).

[2] E. Moreau and O. Vallée, Phys. Rev. E **73**, 016112 (2006).

